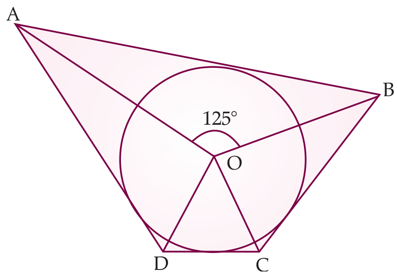
CIRCLES

(A) OBJECTIVE TYPE QUESTIONS — 1 Mark Each

Stand Alone MCQs (1 Mark Each)

1. In the given figure, if $\angle AOB = 125°$, then $\angle COD$ is equal to:

(A) 62.5° **(B)** 45°
(C) 35° **(D)** 55°



U + R

Ans. Option (D) is correct.

Explanation: Since, quadrilateral circumscribing a circle subtends supplementary angles at the centre of the circle.

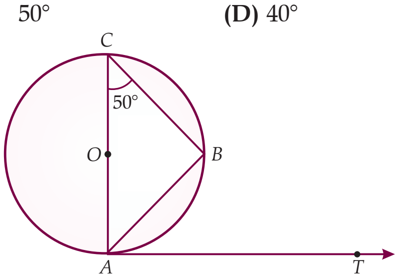
$\therefore \quad \angle AOB + \angle COD = 180°$

$125° + \angle COD = 180°$

$\angle COD = 180° - 125° = 55°$

2. In the given figure, AB is a chord of the circle and AOC is its diameter, such that $\angle ACB = 50°$. If AT is the tangent to the circle at the point A, then $\angle BAT$ is equal to:

(A) 65° **(B)** 60°
(C) 50° **(D)** 40°



U

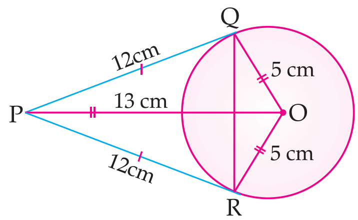
Ans. Option (C) is correct.

Explanation: Since, the angle between chord and tangent is equal to the angle subtended by the same chord in alternate segment of circle.

$\Rightarrow \quad \angle BAT = 50°.$

3. From a point P which is at a distance of 13 cm from the centre O of a circle of radius 5 cm, the pair of tangents PQ and PR to the circle are drawn. Then the area of the quadrilateral PQOR is:

(A) 60 cm^2 **(B)** 65 cm^2
(C) 30 cm^2 **(D)** 32.5 cm^2



C + U

Ans. Option (A) is correct.

Explanation: PQ is tangent and QO is radius at contact point Q.

$\therefore \quad \angle PQO = 90°$

$\therefore$ By Pythagoras theorem,

$PQ^2 = OP^2 - OQ^2$

$= 13^2 - 5^2$

$= 169 - 25 = 144$

$\Rightarrow \quad PQ = 12 \text{ cm}$

$\therefore \quad \Delta OPQ \cong \Delta OPR$

[*SSS* congruence]

$\therefore$ Area of ΔOPQ = area of ΔOPR

[Since, congruent figures are equal in areas]

Area of quadrilateral $QORP$ = 2 area of ΔOPR

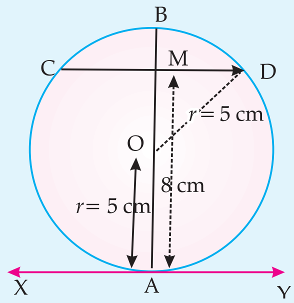
$$= 2 \times \frac{1}{2} \text{ base} \times \text{height}$$
$$= RP \times OR$$
$$= 12 \times 5$$
$$= 60 \text{ cm}^2$$

4. At one end A of diameter AB of a circle of radius 5 cm, tangent XAY is drawn to the circle. The length of the chord CD parallel to XY and at a distance 8 cm from A is:

(A) 4 cm (B) 5 cm
(C) 6 cm (D) 8 cm C + U

Ans. Option (D) is correct.

Explanation: XAY is tangent and AO is radius at contact point A of circle.



$$AO = 5 \text{ cm}$$
$$\therefore \quad \angle OAY = 90^\circ$$

CD is another chord at distance (perpendicular) of 8 cm from A and CMD || XAY meets AB at M.

Join OD.

$$OD = 5 \text{ cm}$$
$$OM = 8 - 5 = 3 \text{ cm}$$
$$\angle OMD = \angle OAY = 90^\circ$$

Now, in right angled ΔOMD

$$MD^2 = OD^2 - MO^2$$
$$= 5^2 - 3^2$$
$$= 25 - 9$$
$$= 16$$
$$\Rightarrow \quad MD = 4 \text{ cm}$$

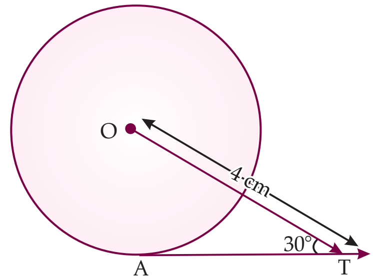
We know that, perpendiculars from centre O of circle bisect the chord.

$$\therefore \quad CD = 2MD$$
$$= 2 \times 4$$
$$= 8 \text{ cm.}$$

Hence, length of chord, $CD = 8$ cm.

5. In the given figure, AT is a tangent to the circle with centre 'O' such that $OT = 4$ cm and $\angle OTA = 30^\circ$. Then AT is equal to:

(A) 4 cm (B) 2 cm
(C) $2\sqrt{3}$ cm (D) $4\sqrt{3}$ cm



C

Ans. Option (C) is correct.

Explanation: Join OA. OA is radius and AT is tangent at contact point A.

$$\therefore \quad \angle OAT = 90^\circ,$$

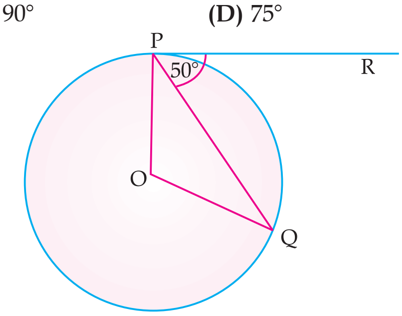
Given that, $OT = 4$ cm

Now, $$\frac{AT}{4} = \frac{\text{base}}{\text{hypotenuse}} = \cos 30^\circ$$

$$\Rightarrow \quad AT = 4 \times \frac{\sqrt{3}}{2} = 2\sqrt{3} \text{ cm.}$$

6. In the given figure, 'O' is the centre of circle, PQ is a chord and the tangent PR at P makes an angle of 50° with PQ, then $\angle POQ$ is equal to:

(A) 100° (B) 80°
(C) 90° (D) 75°



U

Ans. Option (A) is correct.

Explanation: OP is radius and PR is tangent at P.

So, $$\angle OPR = 90^\circ$$
$$\Rightarrow \quad \angle OPQ + 50^\circ = 90^\circ$$
$$\Rightarrow \quad \angle OPQ = 90^\circ - 50^\circ$$
$$\Rightarrow \quad \angle OPQ = 40^\circ$$

In ΔOPQ, $OP = OQ$

[Radii of same circle]

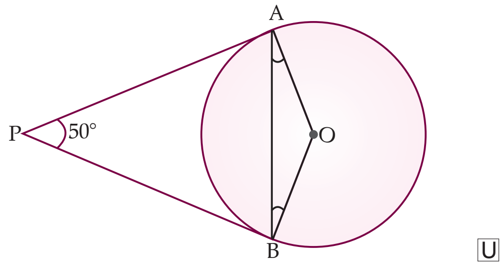
$$\therefore \quad \angle Q = \angle OPQ = 40^\circ$$

[Angles opposite to equal sides are equal]

But, $$\angle POQ = 180^\circ - \angle P - \angle Q$$
$$= 180^\circ - 40^\circ - 40^\circ$$
$$= 180^\circ - 80^\circ = 100^\circ$$
$$\Rightarrow \quad \angle POQ = 100^\circ$$

AI **7. In the given figure, if PA and PB are tangents to the circle with centre O such that $\angle APB = 50^\circ$, then $\angle OAB$ is equal to:**

(A) 25° (B) 30°
(C) 40° (D) 50°



U

Ans. Option (A) is correct.

Explanation: In ΔOAB, we have

$$OA = OB$$

[Radii of same circle]

$\therefore \quad \angle OAB = \angle OBA$

[Angles opposite to equal sides are equal]

As OA and PA are radius and tangent respectively at contact point A.

So, $\quad \angle OAP = 90°$.

Similarly, $\quad \angle OBP = 90°$

Now, in quadrilateral PAOB,

$$\angle P + \angle A + \angle O + \angle B = 360°$$

$\Rightarrow 50° + 90° + \angle O + 90° = 360°$

$\Rightarrow \quad \angle O = 360° - 90° - 90° - 50°$

$\Rightarrow \quad \angle O = 130°$

Again, in ΔOAB,

$$\angle O + \angle OAB + \angle OBA = 180°$$

$\Rightarrow 130° + \angle OAB + \angle OAB = 180°$

$[\because \angle OBA = \angle OAB]$

$\Rightarrow \quad 2\angle OAB = 180° - 130° = 50°$

$\Rightarrow \quad \angle OAB = 25°$

Hence, $\quad \angle OAB = 25°$

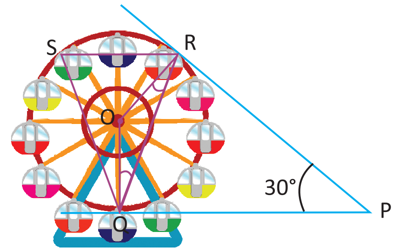
Case-based MCQs (1 Mark Each)

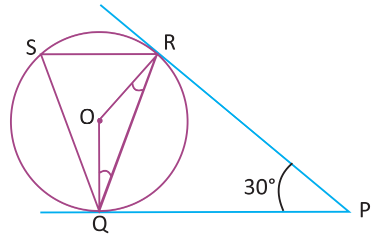
Attempt any four sub-parts from each question. Each sub-part carries 1 mark.

I. Read the following text and answer the questions that follows on the basis of the same: U

A Ferris wheel (or a big wheel in the United Kingdom) is an amusement ride consisting of a rotating upright wheel with multiple passenger-carrying components (commonly referred to as passenger cars, cabins, tubs, capsules, gondolas, or pods) attached to the rim in such a way that as the wheel turns, they are kept upright, usually by gravity.

After taking a ride in Ferris wheel, Aarti came out from the crowd and was observing her friends who were enjoying the ride. She was curious about the different angles and measures that the wheel will form. She forms the figure as given below.





[CBSE QB, 2021]

1. In the given figure find ∠ROQ.

(A) 60° **(B)** 100°

(C) 150° **(D)** 90°

Ans. Option (C) is correct.

Explanation: $\angle ORP = 90° = \angle OQP$

[∵ radius of circle is perpendicular to tangent]

$\therefore \angle ROQ + \angle ORP + \angle OQP + \angle QPR = 360°$

$$\angle ROQ + 90° + 90° + 30° = 360°$$

$$\angle ROQ + 210° = 360°$$

$$\angle ROQ = 360° - 210°$$

$$\angle ROQ = 150°$$

2. Find ∠RQP.

(A) 75° **(B)** 60°

(C) 30° **(D)** 90°

Ans. Option (A) is correct.

Explanation: In ΔOQR

$$\angle OQR = \angle ORQ$$

$$\angle ROQ = 150°$$

and $\angle ROQ + \angle OQR + \angle ORQ = 180°$

$$150° + 2\angle ORQ = 180°$$

$$2\angle ORQ = 30°$$

$$\angle ORQ = 15°$$

$\therefore \quad \angle OQR = \angle ORQ = 15°$

Now $\quad \angle RQP = \angle OQP - \angle OQR$

$= 90° - 15°$

$= 75°$

3. Find ∠RSQ.

(A) 60° **(B)** 75°

(C) 100° **(D)** 30°

Ans. Option (B) is correct.

4. Find ∠ORP.

(A) 90° **(B)** 70°

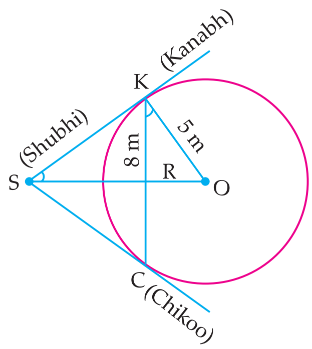
(C) 100° **(D)** 60°

Ans. Option (A) is correct.

> *Explanation:* $\angle ORP = 90°$
> Because, radius of circle is perpendicular to tangent.

AI **II. Read the following text and answer the questions that follows on the basis of the same:**

> There is a circular field of radius 5 m. Kanabh, Chikoo and Shubhi are playing with ball, in which Kanabh and Chikoo are standing on the boundary of the circle. The distance between Kanabh and Chikoo is 8 m. From Shubhi point S, two tangents are drawn as shown in the figure. C + AE



1. What is the relation between the lengths of SK and SC ?

(A) SK ≠ SC (B) SK = SC
(C) SK > SC (D) SK < SC

Ans. Option (B) is correct.

> *Explanation:* We know that the lengths of tangents drawn from an external point to a circle are equal. So, SK and SC are tangents to a circle with centre O.
> $\therefore \quad SK = SC$

2. The length (distance) of OR is:

(A) 3 m (B) 4 m
(C) 5 m (D) 6 m

Ans. Option (A) is correct.

> *Explanation:* In question 1, we have proved
> $$SK = SC$$
> Then ΔSKC is an isosceles triangle and SO is the angle bisector of $\angle$KSC. So, OS $\perp$ KC.
> $\therefore$ OS bisects KC, gives $KR = RC = 4$ cm.
> Now, $OR = \sqrt{OK^2 - KR^2}$
> [By using Pythagoras theorem]
> $= \sqrt{5^2 - 4^2}$
> $= \sqrt{25 - 16}$
> $= \sqrt{9}$
> $= 3$ m.

3. The sum of angles SKR and OKR is:

(A) 45° (B) 30°
(C) 90° (D) None of these

Ans. Option (C) is correct.

> *Explanation:*
> $\angle SKR + \angle OKR = \angle OKS$
> $= 90°$ [Radius is $\perp^r$ to tangent]

4. The distance between Kanabh and Shubhi is:

(A) $\frac{10}{3}$ m (B) $\frac{13}{3}$ m
(C) $\frac{16}{3}$ m (D) $\frac{20}{3}$ m

Ans. Option (D) is correct.

> *Explanation:* ΔSKR and ΔRKO,
> $\angle RKO = \angle KSR$
> and $\angle SRK = \angle ORK$
> $\therefore \quad \Delta KSR \sim \Delta OKR$
> (By AA Similarity)
> Then $\frac{SK}{KO} = \frac{RK}{RO}$
> $\Rightarrow \frac{SK}{5} = \frac{4}{3}$
> [RO = 3 m, proved in Q.2.]
> $\Rightarrow 3SK = 20$
> $\Rightarrow SK = \frac{20}{3}$
> Hence, the distance between Kanabh and Shubhi is $\frac{20}{3}$ m.

5. What is the mathematical concept related to this question ?

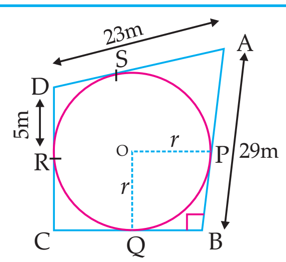
(A) Constructions (B) Area
(C) Circle (D) None of these

Ans. Option (C) is correct.

> *Explanation:* The mathematical concept (**Circle**) is related to this question.

AI **III. Read the following text and answer the questions that follows on the basis of the same:**

> ABCD is a playground. Inside the playground a circular track is present such that it touches AB at point P, BC at Q, CD at R and DA at S.



1. If DR = 5 m, then DS is equal to:

(A) 6 m **(B)** 11 m

(C) 5 m **(D)** 18 m

Ans. Option (C) is correct.

Explanation: DR = 5 m [given]

∴ $DR = DS$

[Length of tangents are equal]

i.e., DS = 5 m.

2. The length of AS is:

(A) 18 m **(B)** 13 m

(C) 14 m **(D)** 12 m

Ans. Option (A) is correct.

Explanation: We have AD = 23 m.

and DS = 5 m (Proved in Q.1)

∴ $AS = AD - DS$

= (23 – 5) m = 18 m.

3. The length of PB is:

(A) 12 m **(B)** 11 m

(C) 13 m **(D)** 20 m

Ans. Option (B) is correct.

Explanation: We have,

AB = 29 m

But AS = AP [Lengths of tangents are equal]

and AS = 18 m [Proved in Q. 2]

∴ AP = 18 m

Now, $PB = AB - AP$

= (29 – 18) m

= 11 m.

4. What is the angle of OQB?

(A) 60° **(B)** 30°

(C) 45° **(D)** 90°

Ans. Option (D) is correct.

Explanation: $\angle OQB$ = 90°

(Radius is $\perp^r$ to tangent)

5. What is the diameter of given circle?

(A) 22 m **(B)** 33 m

(C) 20 m **(D)** 30 m

Ans. Option (A) is correct.

Explanation: ∵ PB = 11 m [proved in Q. 3]

But $PB = BQ$ [Lengths of tangents are equal]

∴ BQ = 11 m

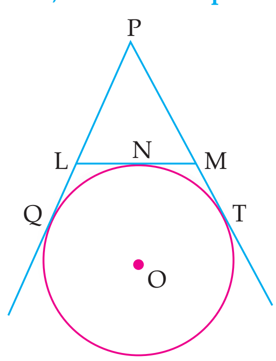
or $r = OQ = QB$ = 11 m

Hence, diameter = $2r$ = 2 × 11 = 22 m.

(B) SUBJECTIVE QUESTIONS

Very Short Answer Type Questions (1 Mark Each)

1. If PQ = 28 cm, then find the perimeter of ΔPLM.



A [CBSE SQP, 2020-21]

Sol. ∵ $PQ = PT$

$PL + LQ = PM + MT$

$PL + LN = PM + MN$

($LQ = LN$, $MT = MN$)

(Tangents to a circle from a common point)

Perimeter (ΔPLM) = $PL + LM + PM$ ½

= $PL + LN + MN + PM$

= 2($PL + LN$)

= 2($PL + LQ$)

= 2 × 28 = 56 cm ½

[CBSE SQP Marking Scheme, 2020-21]

Detailed Solution:

Given, PQ = 28 cm

∴ $PQ = PT$

(Length of tangents from an external point are equal)

i.e., $PQ = PT$ = 28 cm

According to figure,

Let $LQ = x$, then

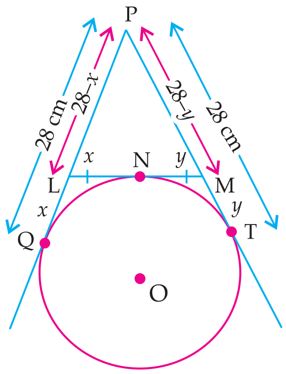
PL = (28 – x) cm

and let $MT = y$, then

PM = (28 – y) cm

and $LM = LN + NM$

= $x + y$



Now, the perimeter of $\Delta PLM = PL + LM + PM$

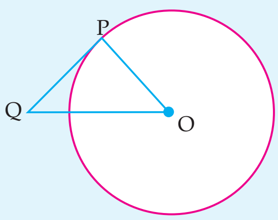
$= (28 - x) + (x + y) + (28 - y)$

$= 28 + 28 = 56$ cm.

AI **2. PQ is a tangent to a circle with centre O at point P. If ΔOPQ is an isosceles triangle, then find ∠OQP.**

A **[CBSE SQP, 2020-21]**

Sol.



In ΔOPQ,

$\angle P + \angle Q + \angle O = 180°$

($\angle O = \angle Q$ isosceles triangle)

$2\angle Q + \angle P = 180°$

$2\angle Q + 90° = 180°$

$2\angle Q = 90°$

$\angle Q = 45°$ **1**

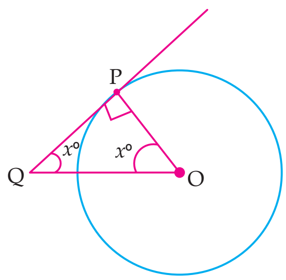
[CBSE SQP Marking Scheme, 2020-21]

Detailed Solution:

$\angle OPQ = 90°$ (Angle between tangent and radius)

Let ∠PQO be $x°$, then

$\angle QOP = x°$ [ΔOPQ is isosceles triangle]



In ΔOPQ,

$\angle OPQ + \angle PQO + \angle QOP = 180°$

[Sum of angles of a triangle]

$\therefore$ $90° + x° + x° = 180°$

$\Rightarrow$ $2x° = 180° - 90° = 90°$

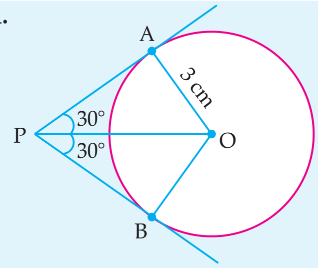
$\Rightarrow$ $x° = \dfrac{90°}{2} = 45°$

Hence, ∠OQP is 45°.

3. If two tangents inclined at 60° are drawn to a circle of radius 3 cm, then find length of each tangent.

A **[CBSE SQP, 2020-21]**

Sol.



In ΔPAO,

$$\tan 30° = \frac{AO}{PA}$$

(Using trigonometry) ½

$$\frac{1}{\sqrt{3}} = \frac{3}{PA}$$

$$PA = 3\sqrt{3} \text{ cm.}$$ ½

[CBSE SQP Marking Scheme, 2020-21]

Detailed Solution:

$PA = PB = ?$

Angle between tangents $= 60°$ (Given)

$\therefore$ Tangents are equally inclined to each other.

$\Rightarrow$ $\angle OPA = \angle OPB = 30°$

and $\angle OAP = 90°$

[Angle between tangent and radius]

In ΔPAO,

$$\tan 30° = \frac{\text{Perpendicular}}{\text{Base}} = \frac{OA}{AP}$$

$\Rightarrow$ $\dfrac{1}{\sqrt{3}} = 3\sqrt{3}$

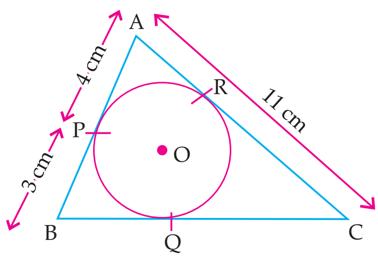
(Using trigonometric Ratios)

$\Rightarrow$ $AP = 3\sqrt{3}$

Hence, the length of each tangent is $3\sqrt{3}$ cm.

AI **4. In the adjoining figure, if ΔABC is circumscribing a circle, then find the length of BC.**

U **[CBSE Delhi Set-I, 2020]**



Sol. $\because$ AP and AR are tangents to the circle from external point A.

$\therefore \quad AP = AR = 4$ cm

Similarly, PB and BQ are tangents.

$\therefore \quad BP = BQ = 3$ cm

Now, $\quad CR = AC - AR = 11 - 4 = 7$ cm

Similarly, CR and CQ are tangents.

$\therefore \quad CR = CQ = 7$ cm

Now, $\quad BC = BQ + CQ = 3 + 7 = 10$ cm.

Hence, the length of BC is 10 cm.

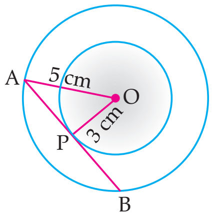
COMMONLY MADE ERROR

- Some students were not versed with the properties of circle.

ANSWERING TIP

- It is necessary for the students to learn all properties of circle.

AI 5. In the given figure, find the length of PB.



U **[CBSE OD Set-I, 2020]**

Sol. Since AB is a tangent at P and OP is radius.

$\therefore \quad \angle APO = 90°, AO = 5$ cm and $OP = 3$ cm

In right angled ΔOPA,

$$AP^2 = AO^2 - OP^2$$

(By using Pythagoras theorem)

$$AP^2 = (5)^2 - (3)^2 = 25 - 9 = 16$$

$\Rightarrow \quad AP = 4$ cm

$\therefore$ Perpendicular from centre to chord bisect the chord

$\therefore \quad AP = BP = 4$ cm.

6. If the radii of two concentric circles are 4 cm and 5 cm, then find the length of each chord of one circle which is tangent to the other circle.

[CBSE SQP, 2020]

Sol. Length of Tangent $= 2 \times \sqrt{5^2 - 4^2}$

$= 2 \times 3$ cm $= 6$ cm ½+½

[CBSE SQP Marking Scheme, 2020]

Detailed Solution:

In ΔOBC,

$$CO^2 + BC^2 = OB^2$$

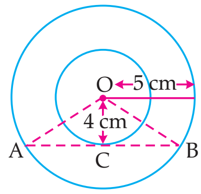
$$4^2 + BC^2 = 5^2$$

$$16 + BC^2 = 25$$

$$BC^2 = 25 - 16$$

$$BC^2 = 9$$

$$BC = 3$$



In ΔOAC,

$$OC^2 + AC^2 = OA^2$$

$$4^2 + AC^2 = 5^2$$

$$AC^2 = 9$$

$$AC = 3$$

$\therefore \quad AB = AC + BC$

$= 3 + 3$

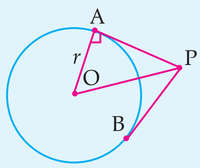
$= 6$ cm.

AI 7. If the angle between two tangents drawn from an external point 'P' to a circle of radius 'r' and centre O is 60°, then find the length of OP.

[CBSE SQP, 2020]

[CBSE Foreign Set-I, II, III, 2016]

Sol.



In OBP, $\quad \dfrac{OB}{OP} = \sin 30°$ ½

$\therefore \quad OP = 2r$ ½

[CBSE Marking Scheme, 2020]

Detailed Solution:

$$OA = r$$

$$OP = ?$$

Angle between tangents $= 60°$

Tangents are equally inclined to each other

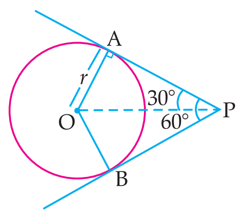
$\Rightarrow \quad \angle OPA = \angle OPB = 30°$

In ΔOPA,

$$\angle POA = 180° - 90° - 30°$$

$$= 60°$$

$\because \quad \cos 60° = \dfrac{OA}{OP}$

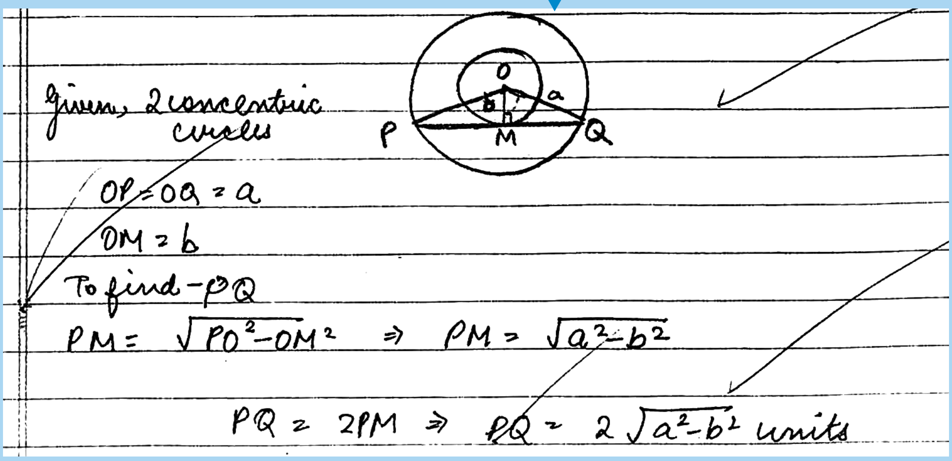


$$\frac{1}{2} = \frac{r}{OP}$$

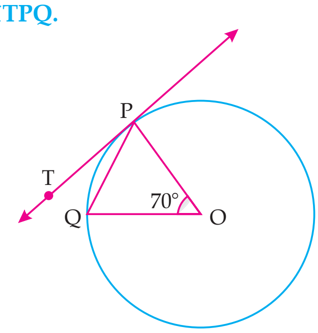
$\Rightarrow \quad OP = 2r$

8. Two concentric circles of radii a and b ($a > b$) are given. Find the length of the chord of the larger circle which touches the smaller circle. [CBSE Delhi Region, 2019]

Topper Answer, 2019



9. In given figure, O is the centre of the circle, PQ is a chord and PT is tangent to the circle at P. Find ∠TPQ.



U [CBSE OD Set-I, II, III, 2017]

Sol. $\angle OPQ = \angle OQP$ (radius of circle)

$$= \frac{180° - 70°}{2} = 55° \quad ½$$

$\therefore$ $\angle TPQ = 90° - 55°$

$= 35°$ ½

[CBSE Marking Scheme, 2017]

Detailed Solution:

According to the figure,

$OP = OQ$ [radii]

$\therefore$ $\angle OPQ = \angle OQP$

[Isosceles triangle property]

Now, in □POQ,

$\angle OPQ + \angle OQP + \angle POQ = 180°$

[Angle sum property]

$\angle OPQ + \angle OPQ + 70° = 180°$

$\angle OPQ = 180° - 70° = 110°$

$\angle OPQ = 55°$

Since $\angle OPT = 90°$ [Angle between tangent and radius]

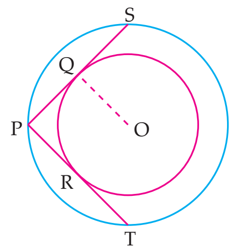
Hence, $\angle TPQ = 90° - \angle OPQ$

$= 90° - 55°$

$= 35°$

10. In the fig. there are two concentric circles with centre O. PRT and PQS are tangents to the inner circle from a point P lying on the outer circle. If $PR = 5$ cm, find the length of PS.

U [Delhi Comptt. Set-I, II, III, 2017]



Sol. $PQ = PR = 5$ cm ½

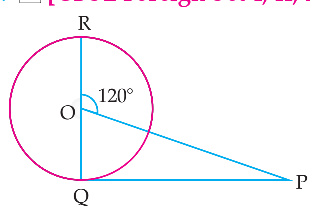
and $PQ = QS$

$\therefore$ $PS = 2PQ$

$= 2 \times 5 = 10$ cm. ½

[CBSE Marking Scheme, 2017]

11. PQ is a tangent drawn from an external point P to a circle with centre O and QOR is the diameter of the circle. If $\angle POR = 120°$, What is the measure of $\angle OPQ$? U [CBSE Foreign Set-I, II, III, 2016, 2017]



Sol. In ΔOQP $\angle POR = \angle OQP + \angle OPQ$

[Exterior angle]

$\therefore$ $\angle OPQ = \angle POR - \angle OQP$

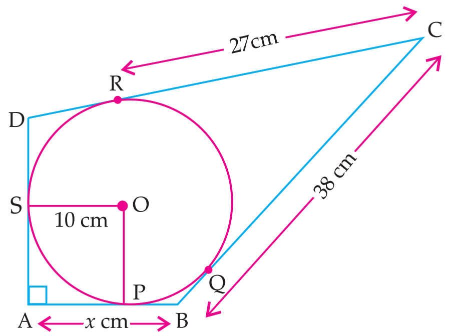
$= 120° - 90°$

$= 30°$

Short Answer Type Questions-I

(2 Marks Each)

1. In the figure, quadrilateral ABCD is circumscribing a circle with centre O and AD ⊥ AB. If radius of incircle is 10 cm, then find the value of x.



A [CBSE SQP, 2020-21]

Sol. $\angle A = \angle OPA = \angle OSA = 90°$ ½

Hence, $\angle SOP = 90°$

Also, $AP = AS$

Hence, OSAP is a square.

$AP = AS = 10$ cm ½

$CR = CQ = 27$ cm

$BQ = BC - CQ$

$= 38 - 27 = 11$ cm ½

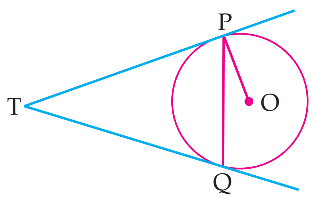
$BP = BQ = 11$ cm

$x = AB = AP + BP$

$= 10 + 11 = 21$ cm ½

[CBSE Marking Scheme, 2020-21]

AI **2. In the given figure, two tangents TP and TQ are drawn to circle with centre O from an external point T. Prove that $\angle PTQ = 2\angle OPQ$.**



U [CBSE Delhi Set-I, 2020]

[CBSE Delhi Set-I, II, III, 2017]

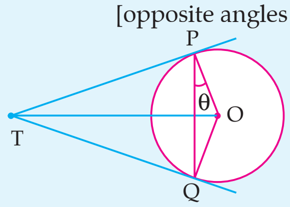
Sol. Let $\angle OPQ$ be θ, then

$\angle TPQ = 90° - \theta$ ½

Since, $TP = TQ$

∴ $\angle TQP = 90° - \theta$ ½

[opposite angles of equal sides]



Now, $\angle TPQ + \angle TQP + \angle PTQ = 180°$ ½

[Angle sum property of a triangle]

$\Rightarrow$ $90° - \theta + 90° - \theta + \angle PTQ = 180°$

$\Rightarrow$ $\angle PTQ = 180° - 180° + 2\theta$

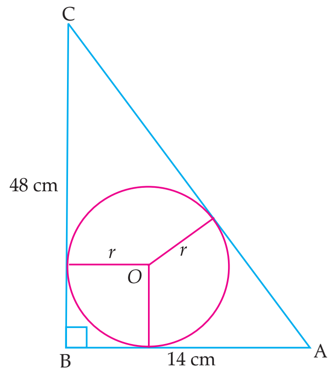
$\Rightarrow$ $\angle PTQ = 2\theta$

Hence, $\angle PTQ = 2\angle OPQ$ ½

Hence Proved.

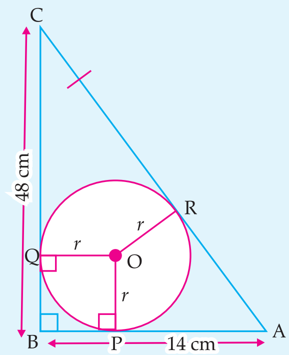
[CBSE Marking Scheme, 2020]

3. In fig. ABC is a triangle in which $\angle B = 90°$, $BC = 48$ cm and $AB = 14$ cm. A circle is inscribed in the triangle, whose centre is O. Find radius of in circle.



A [CBSE Comptt. Set-I, II, III, 2018]

Sol. $AC = \sqrt{AB^2 + BC^2}$



$= \sqrt{14^2 + 48^2} = \sqrt{2500} = 50$ cm ½

$\angle OQB = 90° \Rightarrow OPBQ$ is a square

$BQ = r$, $QC = 48 - r = CR$ ½

Again, $PB = r$

$PA = 14 - r \Rightarrow RA = 48 - r$ ½

$AR + RC = AC \Rightarrow 14 - r + 48 - r = 50$

$r = 6$ cm ½

[CBSE Marking Scheme, 2018]

Detailed Solution:

In ΔABC, $\angle B = 90°$ [Given]

$AC^2 = AB^2 + BC^2$

[By using pythagoras theorem]

$= (14)^2 + (48)^2 = 196 + 2304$

$= 2500$

∴ $AC = \sqrt{2500} = 50$ cm

Here, $\angle OQB = \angle OPB = 90°$

(Radius is perpendicular to tangent)

∴ In Quadrilateral OPBQ,

$\angle POQ = 360° - (OQB + \angle OPB + \angle PBQ)$

$= 360° - (90° + 90° + 90°) = 90°$

So, OPBQ is a square.

Then $OP = QB = BP = OQ = r$

Thus, $CQ = BC - QB = 48 - r$

But $CQ = CR$

[Tangents from external point C]

∴ $CR = 48 - r$

and $AP = AB - BP = 14 - r$

But $AP = AR$

[Tangents from external point A]

∴ $AR = 14 - r$

Now $AC = 50$ cm [proved above]

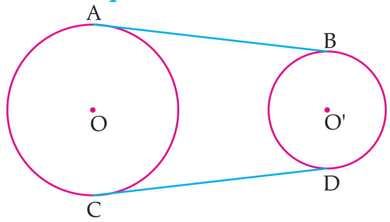
$\Rightarrow$ $AR + RC = 50$

$\Rightarrow \quad 14 - r + 48 - r = 50$

$\Rightarrow \quad -2r = 50 - 62 = -12$

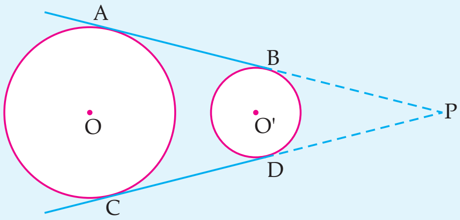
$\Rightarrow \quad r = 6$ cm.

4. In the fig., AB and CD are common tangents to two circles of unequal radii. Prove that $AB = CD$.



A [CBSE Comptt. Delhi Set-III, 2017]

Sol. Construction : Produce AB and CD to meet at P.



Now, $PA = PC$

and $PB = PD$ ½

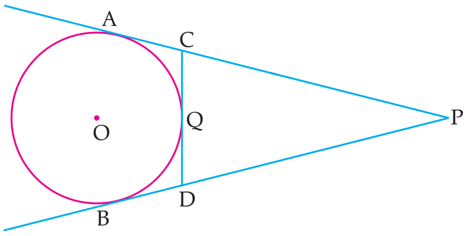
Tangents to a circle from external point ½

Now, $PA - PB = PC - PD$ ½

$\Rightarrow \quad AB = CD$ ½

[CBSE Marking Scheme, 2017]

5. In the given figure, PA and PB are tangents to the circle from an external point P. CD is another tangent touching the circle at Q. If $PA = 12$ cm, $QC = DQ = 3$ cm, then find PC + PD.



A [CBSE Comptt. Delhi Set-I, II, III, 2017]

Sol. Here, $AC = CQ$ [Tangents from external point to a circle]

$PA = PC + CA = PC + CQ$

$[\because \ CA = CQ]$

$\Rightarrow \quad 12 = PC + 3$

$\Rightarrow \quad PC = 12 - 3 = 9$ cm **1**

$PB = PD + BD$

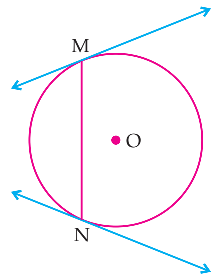
$PA = PD + DQ$

$12 - 3 = PD = 9$ cm

$\therefore \quad PC + PD = 9 + 9 = 18$ cm **1**

[CBSE Marking Scheme, 2017]

6. Prove that the tangents drawn at the end points of a chord of a circle make equal angles with the chord.



A [CBSE, OD Set-I, II, III, 2017]

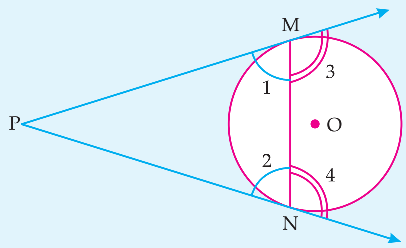
[CBSE Delhi Term-II, 2015]

Sol. $\because PM = PN$ [length of tangents are equal]

$\angle 1 = \angle 2$ [angles opp. to equal sides are equal] **1**

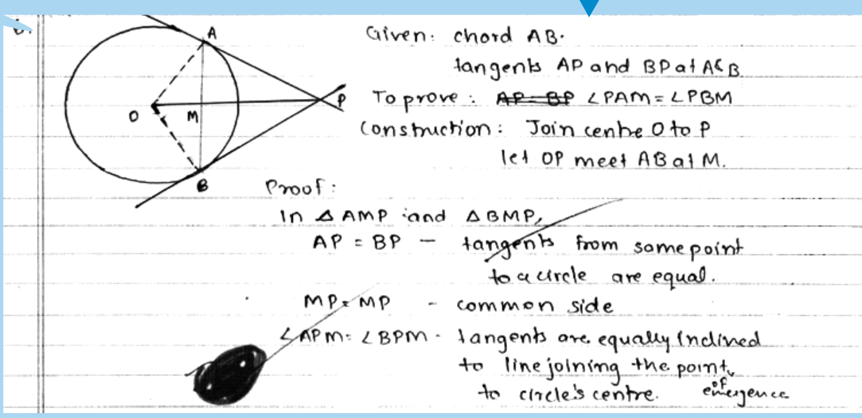
$\because \quad 180° - \angle 1 = 180° - \angle 2$ [linear pair]

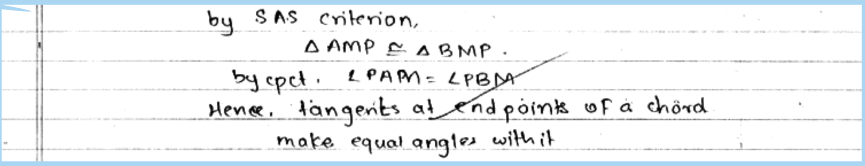
$\angle 3 = \angle 4$ **1**



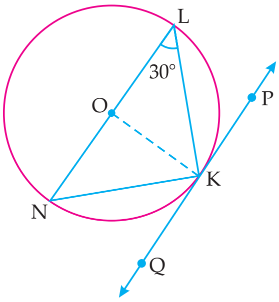
[CBSE Marking Scheme, 2015]

Topper Answer, 2017


Given: chord AB.
tangents AP and BP at A & B.
To prove: ~~AP = BP~~ ∠PAM = ∠PBM
Construction: Join centre O to P
let OP meet AB at M.
Proof:
In △AMP and △BMP,
AP = BP – tangents from same point to a circle are equal.
MP = MP – common side
∠APM = ∠BPM – tangents are equally inclined to line joining the point of emergence to circle's centre.


by SAS criterion,
$\Delta AMP \cong \Delta BMP$.
by cpct. $\angle PAM = \angle PBM$
Hence, tangents at end points of a chord make equal angles with it

7. In given figure, O is the centre of the circle and LN is a diameter. If PQ is a tangent to the circle at K and $\angle KLN = 30°$, find $\angle PKL$.



U **[CBSE Comptt. OD Set-I, II, III, 2017]**

Sol. Here, $OK = OL$ [radii]

$\angle OKL = \angle OLK = 30°$

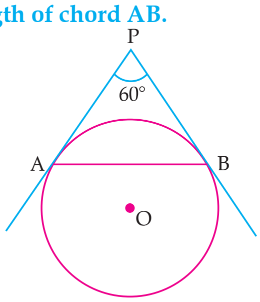
[Opposite angles of equal sides] **1**

Since $\angle OKP = 90°$ [Tangent]

$\therefore$ $\angle PKL = 90° - 30° = 60°$ **1**

[CBSE Marking Scheme, 2017]

8. In fig., AP and BP are tangents to a circle with centre O, such that $AP = 5$ cm and $\angle APB = 60°$. Find the length of chord AB.



U **[CBSE Delhi Set-I, II, III, 2016]**

Sol. $PA = PB$ ½

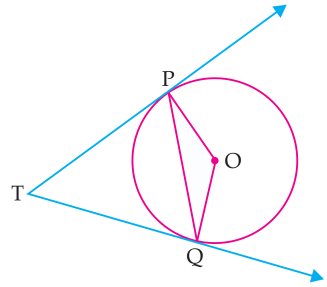
or, $\angle PAB = \angle PBA = 60°$ ½

$\therefore$ ΔPAB is an equilateral triangle. ½

Hence, $AB = PA = 5$ cm. ½

[CBSE Marking Scheme, 2016]

9. In the given figure PQ is chord of length 6 cm of the circle of radius 6 cm. TP and TQ are tangents to the circle at points P and Q respectively. Find $\angle PTQ$.



U **[CBSE SA-II, 2016]**

Sol. Here, $PQ = 6$ cm, $OP = OQ = 6$ cm

$\therefore$ $PQ = OP = OQ$

$\therefore$ $\angle POQ = 60°$

[angle of equilateral Δ]

$\angle OPT = \angle OQT = 90°$

[radius ⊥ tangent]

$\therefore$ $\angle PTQ + 90° + 90° + 60° = 360°$

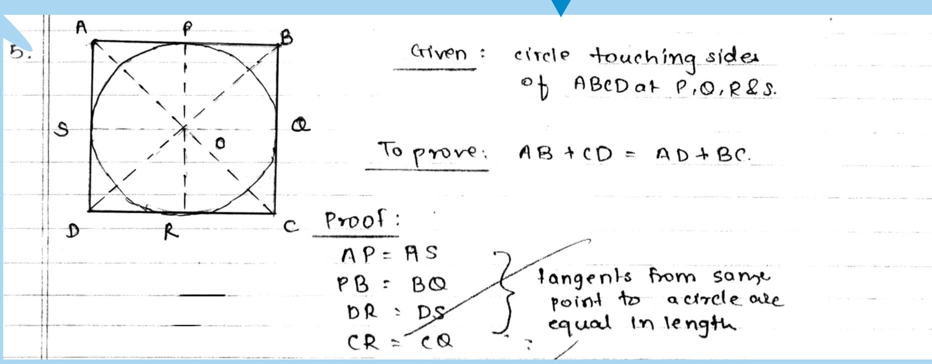
[angle sum property]

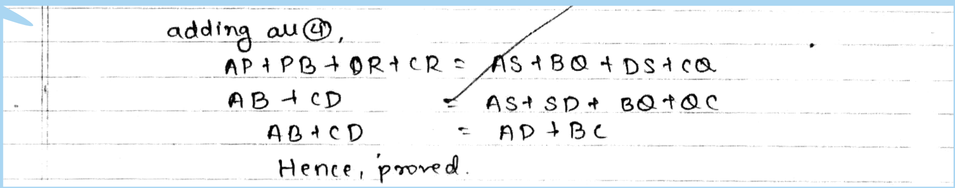
$\angle PTQ = 120°$

10. A circle touches all the four sides of a quadrilateral ABCD. Prove that $AB + CD = BC + DA$.

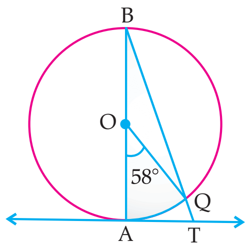
A **[CBSE OD Set-I, II, III, 2016]**

Topper Answer, 2016


5.
Given : circle touching sides of ABCD at P, Q, R & S.
To prove: AB + CD = AD + BC.
Proof :
AP = AS
PB = BQ
DR = DS
CR = CQ
} tangents from same point to a circle are equal in length


adding all ④,
AP + PB + DR + CR = AS + BQ + DS + CQ
AB + CD = AS + SD + BQ + QC
AB + CD = AD + BC
Hence, proved.

11. In given figure, AB is the diameter of a circle with center O and AT is a tangent. If $\angle AOQ = 58°$, find $\angle ATQ$. U **[CBSE Term-II, Set-I, II, III, 2015]**



Sol. $\angle AOQ = 58°$ (Given)

$\angle ABQ = \frac{1}{2} \angle AOQ$

[Angle on the circumference of the circle by the same arc]

$= \frac{1}{2} \times 58°$

$= 29°$ **1**

$\angle BAT = 90°$ [$\because OA \perp AT$]

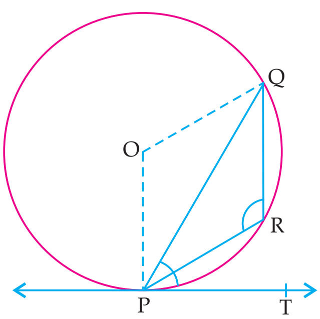
$\therefore$ $\angle ATQ = 90° - 29°$

$= 61°$ **1**

[CBSE Marking Scheme, 2015]

12. In figure, PQ is a chord of a circle centre O and PT is a tangent. If $\angle QPT = 60°$, find $\angle PRQ$.

[CBSE, OD Set-I, II, III, 2015]



Sol. Given, $\angle QPT = 60°$

$\angle OPQ = \angle OQP = 90° - 60° = 30°$

$\angle POQ = 180° - (30° + 30°)$

$= 180° - 60° = 120°$

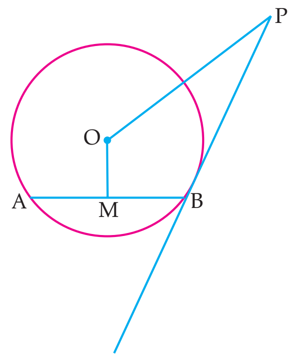
$\angle PRQ = \frac{1}{2}$ Reflex $\angle POQ$ **1**

[$\because$ Reflex $\angle POQ = 360° - 120° = 240°$]

$= \frac{1}{2} \times 240° = 120°$

[CBSE Marking Scheme, 2015] 1

13. PB is a tangent to the circle with centre O to B. AB is a chord of length 24 cm at a distance of 5 cm from the centre. If the tangent is of length 20 cm, find the length of PO.



A **[CBSE Delhi Term-II, 2015]**

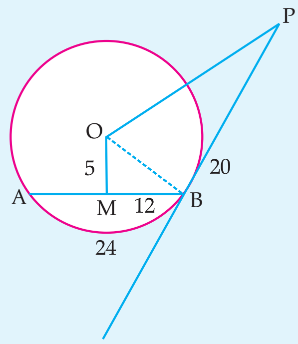
Sol. Construction : Join OB.

In rt. ΔOMB,

$OB^2 = 5^2 + 12^2 = 13^2$

$\therefore$ $OB = 13$ cm **1**

Since OB $\perp$ PB [radius $\perp$ tangent]



$\therefore$ In rt. ΔOBP,

$OP^2 = OB^2 + BP^2$

$= 13^2 + 20^2$

$= 569$

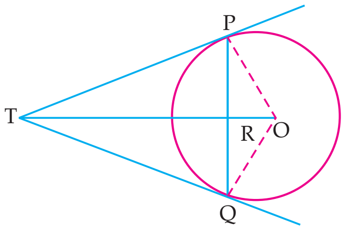
or, $OP = \sqrt{569} = 23.85$ cm. **1**

[CBSE Marking Scheme, 2015]

14. From a point T outside a circle of centre O, tangents TP and TQ are drawn to the circle. Prove that OT is the right bisector of line segment PQ.

A **[CBSE Delhi Term-II, Set-I, II, III, 2015]**

Sol. Given: A circle with centre O. Tangents TP and TQ are drawn from a point T outside a circle.



To Prove: OT is the right bisector of line segment PQ.

Construction: Join OP and OQ

Proof: ΔOPT and ΔOTQ

$PT = PQ$ [Tangents of the circle]

$OT = OT$ [Common side]

$\angle OPT = \angle OQR = 90°$

$\therefore \quad \Delta OPT \cong \Delta OQT$ [R.H.S. Congruency]

$\angle PTO = \angle QTO$ [c.p.c.t]

ΔPTR and ΔTRQ

$TP = TQ$ [Tangents of circle]

$TR = TR$ [Common]

$\Delta PTR \cong \Delta QTR$ [SAS congruency]

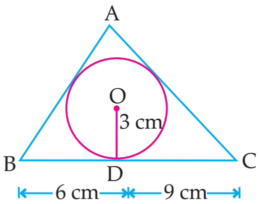
$\angle PRT = \angle TRQ$ [c.p.c.t.]

$PR = QR$ [c.p.c.t.]

$\angle PRT + \angle TRQ = 180°$

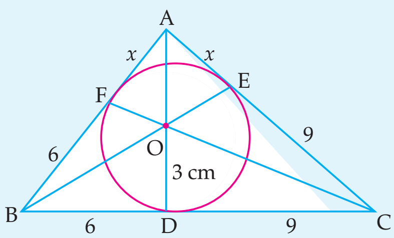
$\therefore \quad \angle PRT = \angle TRQ = 90°$

15. In figure, a triangle ABC is drawn to circumscribe a circle of radius 3 cm, such that the segments BD and DC are respectively of lengths 6 cm and 9 cm. If the area of ΔABC is 54 cm^2, then find the lengths of sides AB and AC.



A [CBSE OD Set-I, II, III, 2015]

Sol.



Let $AF = AE = x$.

$\therefore \quad AB = 6 + x, AC = 9 + x$ and $BC = 15$ ½

$$\text{ar } \Delta ABC = \frac{1}{2}[15 + 6 + x + 9 + x].3 = 54$$

$45 + 3x = 54$ **1**

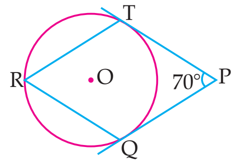
or, $x = 3$

$\therefore \quad AB = 9$ cm, $AC = 12$ cm ½

and $BC = 15$ cm.

[CBSE Marking Scheme, 2015]

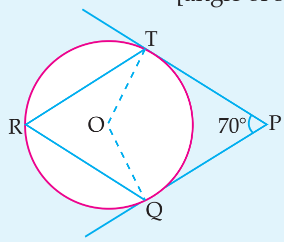
16. In figure, O is the centre of a circle. PT and PQ are tangents to the circle from an external point P. If ∠TPQ = 70°, find ∠TRQ.



U [Foreign Set-I, II, III, 2015]

Sol. $\angle TOQ = 180° - 70° = 110°$ **1**

[angle of supplementary]



Then, $\angle TRQ = \frac{1}{2}\angle TOQ$

[angle at the circumference of the circle by same arc]

$= \frac{1}{2} \times 110° = 55°$ **1**

[CBSE Marking Scheme, 2015]

Short Answer Type Questions-II

(3 Marks Each)

1. Prove that the parallelogram circumscribing a circle is a rhombus. A [CBSE Delhi Set-II, 2020]

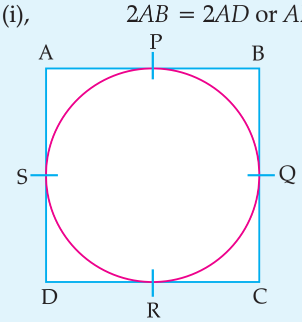
Sol. Let ABCD be the || ^{gm}.

$\therefore \quad AB = CD$ and $AD = BC$ ...(i) ½

$AP + PB + DR + CR = AS + BQ + DS + CQ$ **1**

or, $AB + CD = AD + BC$ ½

From (i), $2AB = 2AD$ or $AB = AD$



or, ABCD is a rhombus. **1**

[CBSE Marking Scheme, 2020]

Detailed Solution:

Let ABCD be the parallelogram.

$\therefore AB = CD$ and $AD = BC$ ...(i)

We know that the tangents drawn to a circle from an exterior point are equal in length.

Therefore, $AP = AS$, $BP = BQ$, $CR = CQ$ and $DR = DS$.

Adding the above equations.

$$AP + BP + CR + DR = AS + BQ + CQ + DS$$

$$\Rightarrow (AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ)$$

$$\Rightarrow \quad AB + CD = AD + BC$$

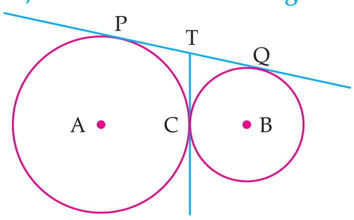
From eq. (i),

$$2AB = 2AD$$

or, $$AB = AD$$

Hence, $ABCD$ is a rhombus. **Hence Proved.**

AI **2. In given fig., two circles touch each other at the point C. Prove that the common tangent to the circles at C, bisects the common tangent at P and Q.**



A **[CBSE Delhi Set-III, 2020]**

Sol. Since, $$PT = TC$$

and $$QT = TC$$

[tangents of circle from external point]

So, $$PT = QT$$

Now $$PQ = PT + TQ$$

$$\Rightarrow \quad PQ = PT + PT$$

$$\Rightarrow \quad PQ = 2PT$$

$$\Rightarrow \quad \frac{1}{2}PQ = PT$$

Hence, the common tangent to the circle at C, bisects the common tangents at P and Q.

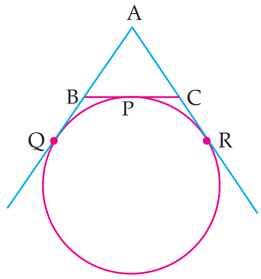
AI **3. If a circle touches the side BC of a triangle ABC at P and extended sides AB and AC at Q and R, respectively, prove that $AQ = \frac{1}{2}(BC + CA + AB)$.**

A **[CBSE OD Set-I, 2020]**

Sol. $BC + CA + AB$

$$= (BP + PC) + (AR - CR) + (AQ - BQ)$$

$$= AQ + AR - BQ + BP + PC - CR$$



$\because$ From the same external point, the tangent segments drawn to a circle are equal.

From the point B, $BQ = BP$

From the point A, $AQ = AR$

From the point C, $CP = CR$

$\therefore$ Perimeter of ΔABC,

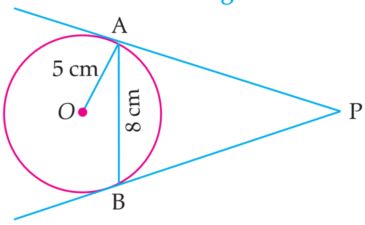
$$AB + BC + CA = 2AQ - BQ + BQ + CR - CR]$$

$$\Rightarrow \quad = 2AQ$$

$$\Rightarrow \quad AQ = \frac{1}{2}(BC + CA + AB)$$

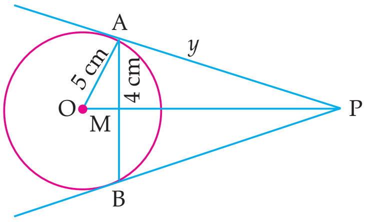
Hence proved.

4. In figure AB is a chord of length 8 cm of a circle of radius 5 cm. The tangents to the circle at A and B intersect at P. Find the length of AP.



A **[CBSE Delhi Set-I, 2019, Comptt. Set-I, II, III, 2018]**

Sol.



Given, $$AB = 8 \text{ cm}$$

$$\Rightarrow \quad AM = 4 \text{ cm.}$$

$$\therefore \quad OM = \sqrt{OA^2 - AM^2}$$

[By Pythagoras theorem]

$$OM = \sqrt{5^2 - 4^2} = 3 \text{ cm.}$$

Let $AP = y$ cm, $PM = x$ cm.

$\because$ ΔOAP is a right angle triangle.

$$\therefore \quad OP^2 = OA^2 + AP^2$$

[By Pythagoras theorem]

$$(x + 3)^2 = y^2 + 25$$

$$\Rightarrow \quad x^2 + 9 + 6x = y^2 + 25 \quad \text{...(i)}$$

Also, $$x^2 + 4^2 = y^2 \quad \text{...(ii)}$$

$$x^2 + 6x + 9 = x^2 + 16 + 25$$

$$6x = 32$$

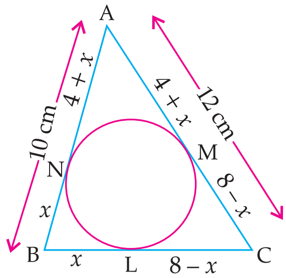
$$\Rightarrow \quad x = \frac{32}{6} \text{ or } \frac{16}{3} \text{ cm}$$

$$y^2 = x^2 + 16 = \frac{256}{9} + 16$$

$$= \frac{400}{9}$$

$$y = \frac{20}{3} \text{ cm or } 6\frac{2}{3} \text{ cm.}$$

AI **5. In the given figure a circle is inscribed in a ΔABC having sides $BC = 8$ cm, $AB = 10$ cm and $AC = 12$ cm. Find the length BL, CM and AN.**



Ⓐ **[CBSE Delhi Set-II, 2019]**
[Delhi Set-I, II, III, 2016]

Sol. Let $BL = x = BN$

[Tangents from external point B]

$\therefore \quad CL = 8 - x = CM$

[Tangents from external point C]

$\because \quad AC = 12$

$\Rightarrow \quad AM = 4 + x = AN$ **1**

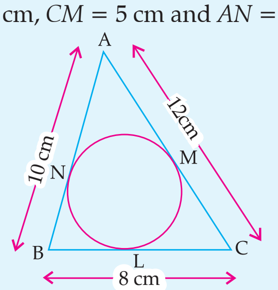
[Tangents from external point A]

Now $AB = AN + NB = 10$

$\Rightarrow \quad x + 4 + x = 10$

$\Rightarrow \quad x = 3$ **1**

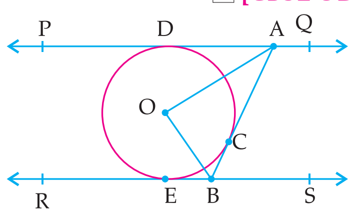
$\therefore$ $BL = 3$ cm, $CM = 5$ cm and $AN = 7$ cm. **1**



6. In figure PQ and RS are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting PQ at A and RS at B. Prove that $\angle AOB = 90°$.

[CBSE Delhi Set-I, II, III, 2017]

Ⓐ **[CBSE OD Set-I, 2019]**



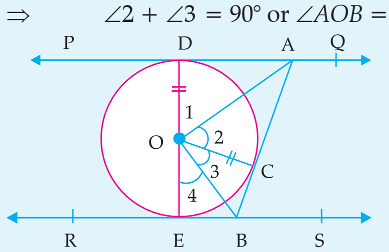
Sol. $\Delta AOD \cong \Delta AOC$ [SAS] **1**

$\Rightarrow \quad \angle 1 = \angle 2$ ½

Similarly, $\quad \angle 4 = \angle 3$ ½

$\Rightarrow \quad \angle 1 + \angle 4 = \angle 2 + \angle 3 = \frac{1}{2}(180°)$

$\Rightarrow \quad \angle 2 + \angle 3 = 90°$ or $\angle AOB = 90°$ ½



½

Alternate method :

$\Delta OAD \cong \Delta AOC$ (By SAS)

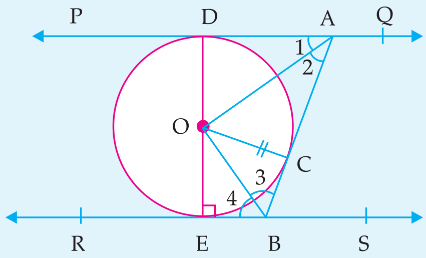
$\Rightarrow \quad \angle 1 = \angle 2$ **1**

Similarly, $\quad \angle 4 = \angle 3$ ½

But $\angle 1 + \angle 2 + \angle 3 + \angle 4 = 180°$ $(\because PQ \parallel RS)$

$\Rightarrow \quad \angle 2 + \angle 3 = \angle 1 + \angle 4 = \frac{1}{2}(180°) = 90°$

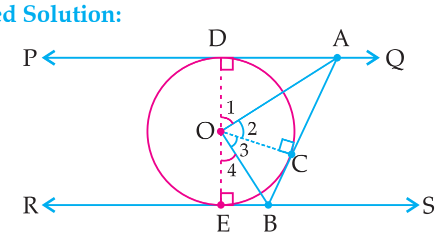
$\therefore$ In ΔAOB, $\angle AOB = 180° - (\angle 2 + \angle 3) = 90°$ ½



½

[CBSE Marking Scheme, 2019]

Detailed Solution:



In $\Delta\, DOA$ and $\Delta\, COA$

$DA = AC$

[Tangents drawn from common point]

$\angle ODA = \angle OCA = 90°$

[angle between tangent and radius]

$OD = OC$ [radius of circle]

$\therefore \quad \Delta DOA \cong \Delta COA$ [By SAS]

Hence, $\angle 1 = \angle 2$ *i.e.*, $\angle DOA = \angle COA$ [By cpct] ...(i)

Similarly,

$\Delta\, BOC \cong \Delta\, BOE$ [By SAS]

$\therefore \quad \angle 3 = \angle 4$ *i.e.*, $\angle COB = \angle BOE$ [By cpct] ...(ii)

Now, $\angle 1 + \angle 2 + \angle 3 + \angle 4 = 180°$

[angles on a straight line]

$2\angle 2 + 2\angle 3 = 180°$

[from eq. (i) & (ii)]

$\angle 2 + \angle 3 = 90°$

i.e., $\quad \angle AOC + \angle BOC = 90°$

or $\quad \angle AOB = 90°$ **Hence Proved.**

COMMONLY MADE ERROR

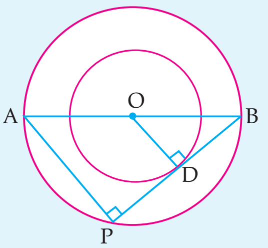
- Some candidates could not apply the appropriate theorem to find out the unknown angles.

ANSWERING TIP

- Learn circle and related angle properties, cyclic properties, tangent and secant properties thoroughly.

7. The radii of two concentric circles are 13 cm and 8 cm. AB is a diameter of the bigger circle and BD is a tangent to the smaller circle touching it at D and intersecting the larger circle at P on producing. Find the length of AP. U **[CBSE SQP, 2018-19]**

Sol.



1

	$\angle APB$	$= 90°$	[angle in semi-circle]
and	$\angle ODB$	$= 90°$	[radius is perpendicular to tangent]
	ΔABP	$\sim \Delta OBD$	

$$\Rightarrow \quad \frac{AB}{OB} = \frac{AP}{OD} \qquad \textbf{1}$$

$$\Rightarrow \quad \frac{26}{13} = \frac{AP}{8}$$

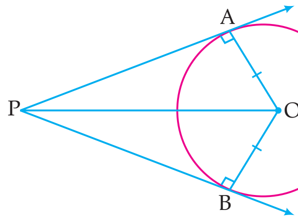
Hence, $AP = 16$ cm **1**

[CBSE Marking Scheme, 2018-19]

8. Prove that the lengths of two tangents drawn from an external point to a circle are equal.

A **[CBSE OD Set-I, II, III, 2018]**

Sol. Given: AP and BP are tangents of circle having centre O.



To Prove : $AP = BP$

Construction : Join OP, AO and BO

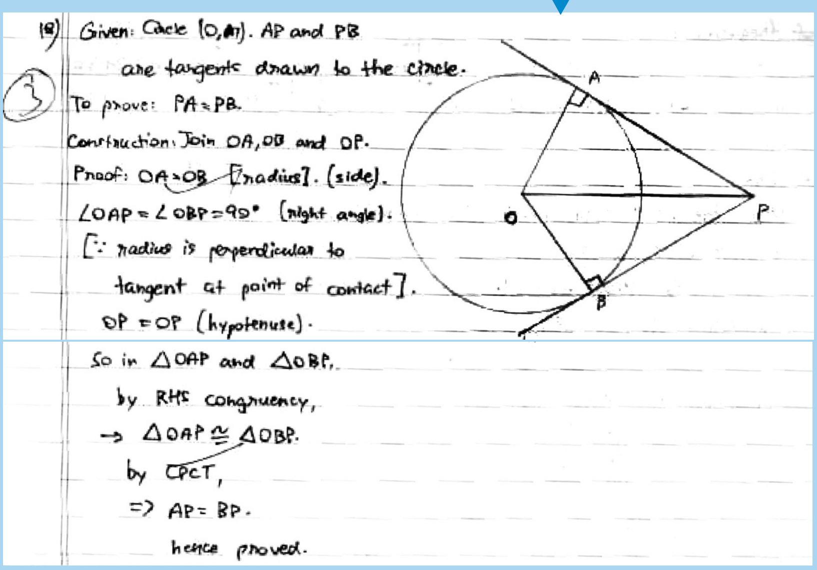
Proof : ΔOAP and ΔOBP

$OA = OB$		[Radius of circle]
$OP = OP$		[Common side]
$\angle OAP = \angle OBP = 90°$		[Radius ⊥ tangent]
$\Delta OAP \cong \Delta OBP$		[By RHS]
$AP = BP$		[By cpct]

Hence Proved.

Detailed Solution:

Topper Answer, 2018


(18) Given: Circle (O, r). AP and PB are tangents drawn to the circle.

To prove: PA = PB.

Construction: Join OA, OB and OP.

Proof: OA = OB [radius]. (side).

∠OAP = ∠OBP = 90° (right angle).

[∵ radius is perpendicular to tangent at point of contact].

OP = OP (hypotenuse).

So in ΔOAP and ΔOBP,

by RHS congruency,

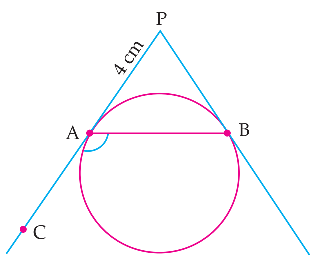
→ ΔOAP ≅ ΔOBP.

by CPCT,

⇒ AP = BP.

hence proved.

9. In the given figure, PA and PB are tangents to a circle from an external point P such that $PA = 4$ cm and $\angle BAC = 135°$. Find the length of chord AB.



U **[CBSE OD Set I, II, III, 2017]**

Sol. $PA = PB = 4$ cm

[Tangents from external point] ½

$\angle PAB = 180° - 135° = 45°$

[Supplementary angles]

$\angle ABP = \angle PAB = 45°$

[Opposite angles of equal sides] ½

$\therefore \quad \angle APB = 180° - 45° - 45°$

$= 90°$

So, ΔABP is an isosceles right angled triangle.

$\Rightarrow \quad AB^2 = 2AP^2$ **1**

$\Rightarrow \quad AB^2 = 32$ **1**

Hence, $\quad AB = \sqrt{32} = 4\sqrt{2}$ cm

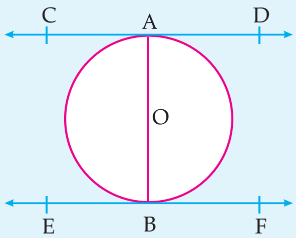
[CBSE Marking Scheme, 2017]

10. Prove that the tangents drawn at the ends of the diameter of a circle are parallel.

A **[CBSE Delhi Set-I, II, III, 2017]**

Sol. Let AB be the diameter of a given circle and let CD and EF be the tangents drawn to the circle at A and *B* respectively.

AB ⊥ CD and AB ⊥ EF **1**



$\therefore \quad \angle CAB = 90°$ and $\angle ABF = 90°$ ½

$\angle CAB = \angle ABF$

and $\quad \angle ABE = \angle BAD$ ½

Hence, $\angle CAB$ and $\angle ABF$ also $\angle ABE$ and $\angle BAD$ are alternate interior angles. **1**

$\therefore \quad CD \parallel EF$ **Hence proved.**

[CBSE Marking Scheme, 2017]

11. ABC is a triangle. A circle touches sides AB and AC produced and side BC at X, Y and Z respectively. Show that

$AX = \dfrac{1}{2}$ **perimeter of** ΔABC.

A **[CBSE Term-II, 2016]**

Sol. Try yourself like Q. 3. SATQ-II.

Long Answer Type Questions

(5 Marks Each)

1. Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

A **[CBSE Delhi Region, 2019]**

[CBSE Foreign Set-I, II, III, 2017]

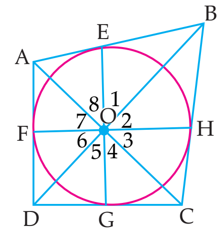
Sol. Given: A circle with centre O is inscribed in a quadrilateral ABCD.

In ΔAEO and ΔAFO,

$OE = OF$ [radii of circle]

$\angle OEA = \angle OFA = 90°$

[radius is $\perp^r$ to tangent] **1**



1

The point of contact is perpendicular to the tangent.

$OA = OA$ [common side]

$\Delta AEO \cong \Delta AFO$

[By RHS]

$\angle 7 = \angle 8$ (By cpct) ...(i) **1**

Similarly,

$\angle 1 = \angle 2$ ...(ii)

$\angle 3 = \angle 4$ ...(iii)

$\angle 5 = \angle 6$ ...(iv)

$\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360°$ **1**

[angle around a point]

$2\angle 1 + 2\angle 8 + 2\angle 4 + 2\angle 5 = 360°$

$\angle 1 + \angle 8 + \angle 4 + \angle 5 = 180°$

$(\angle 1 + \angle 8) + (\angle 4 + \angle 5) = 180°$ **1**

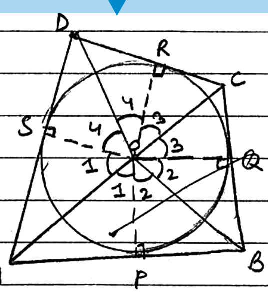
$\angle AOB + \angle COD = 180°$

Hence Proved.

Topper Answer, 2018

To prove: opposite sides of a quadrilateral circumscribing a circle subtend equal angles at the centre.

Construction: Constructed



a quadrilateral ABCD, circumscribing a circle (centre O).
Circle touches AB, BC, CD, DA at P, Q, R, S respectively.

To prove : ∠AOB + ∠COD = 180°
Or ∠AOD + ∠BOC = 180°

We know, that tangents from same exterior point subtend equal angle at the centre of circle with radius.

∴ ∠AOP = ∠AOS = ∠1 (say)

Similarly, ∠BOP = ∠BOQ = ∠2
∠COQ = ∠COR = ∠3
∠DOR = ∠DOS = ∠4.

∴ ∠AOP + ∠BOP + ∠BOQ + ∠COQ + ∠COR + ∠DOR + ∠DOS + ∠AOS = 360°
[Complete angle around a point]

⇒ 2∠1 + 2∠2 + 2∠3 + 2∠4 = 360°
⇒ ∠1 + ∠2 + ∠3 + ∠4 = 180°
⇒ (∠1 + ∠2) + (∠3 + ∠4) = 180°.
⇒ ∠AOB + ∠COD = 180°

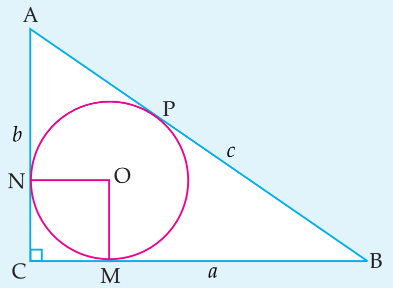
Or (∠1 + ∠4) + (∠2 + ∠3) = 180°
⇒ ∠AOD + ∠BOC = 180°

Hence, proved!

2. ***a, b*** **and** ***c*** **are the sides of a right triangle, where** ***c*** **is the hypotenuse. A circle, of radius** ***r*****, touches the sides of the triangle. Prove that,** $r = \frac{a+b-c}{2}$.

A [CBSE Term-II, 2016]

Sol.



1

Let circle touches CB at M, CA at N and AB at P.

Now OM ⊥ CB and ON ⊥ AC [radius ⊥ tangent]

$OM = ON$ [radii]

$CM = CN$ [Tangents] **1**

∴ OMCN is a square.

Let $OM = r = CM = CN$ **1**

$AN = AP$, $CN = CM$ and $BM = BP$

[tangents from external point]

$AN = AP$

⇒ $AC - CN = AB - BP$ **1**

$b - r = c - BM$

$b - r = c - (a - r)$

$b - r = c - a + r$

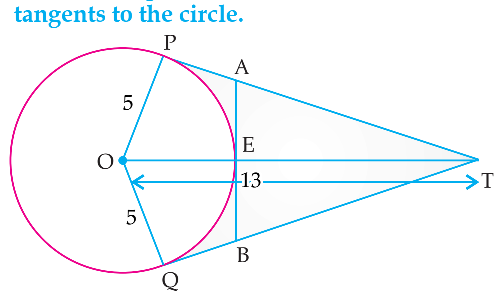
∴ $2r = a + b - c$

$r = \frac{a+b-c}{2}$. **1**

Hence Proved.

[CBSE Marking Scheme, 2016]

3. In Fig. O is the centre of a circle of radius 5 cm. T is a point such that ***OT*** **= 13 cm and OT intersects circle at E. If AB is a tangent to the circle at E, find the length of AB, where TP and TQ are two tangents to the circle.**



U [CBSE Delhi Set-I, II, III, 2016]

Sol. $PT = \sqrt{169-25} = 12$ cm

and $TE = OT - OE = 13 - 5$

$= 8$ cm ½ + ½

Let $PA = AE = x.$ (Tangents)

Then, $TA^2 = TE^2 + EA^2$ **1**

or, $(12-x)^2 = 8^2 + x^2$ **1**

$24x = 80$

or, $x = 3.3$ cm. (Approx.) **1**

Thus $AB = 2 \times x = 2 \times 3.3$

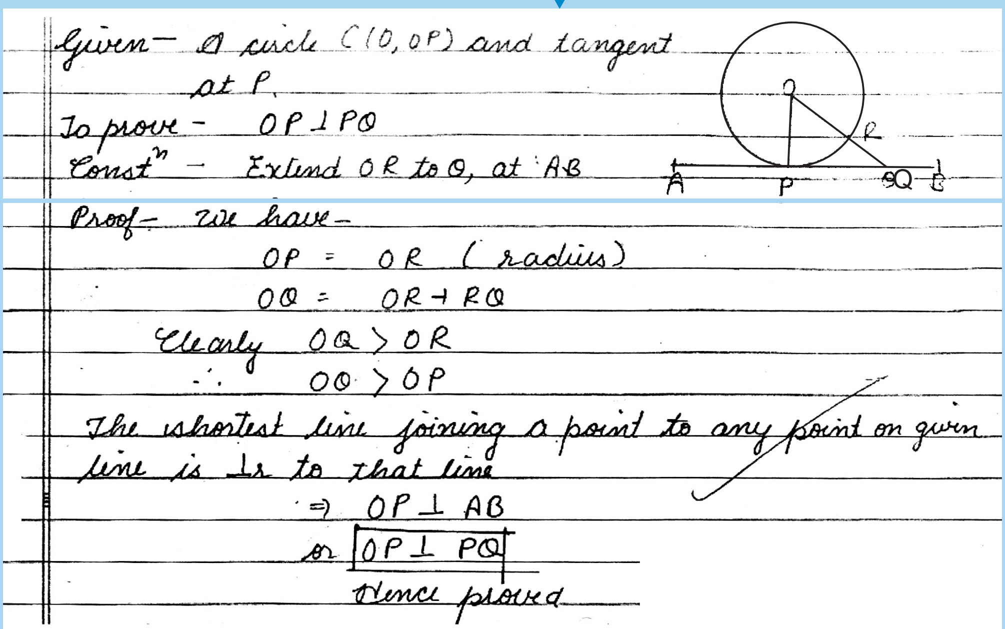
$= 6.6$ cm. (Approx.) **1**

[CBSE Marking Scheme, 2016]

4. Prove that tangent drawn at any point of a circle is perpendicular to the radius through the point of contact.

U **[CBSE OD Set-II, 2016]**

Topper Answer, 2016



Given – A circle C(O, OP) and tangent at P.

To prove – OP ⊥ PQ

Const^n – Extend OR to Q, at AB

Proof – We have –

OP = OR (radius)

OQ = OR + RQ

Clearly OQ > OR

∴ OQ > OP

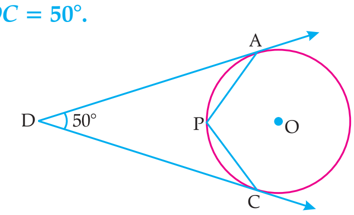
The shortest line joining a point to any point on given line is ⊥r to that line

⇒ OP ⊥ AB

or OP ⊥ PQ

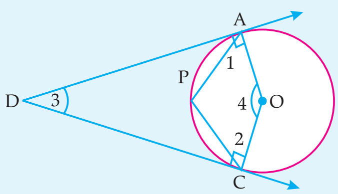
Hence proved

5. In the given figure, O is the centre of the circle. Determine $\angle APC$, if DA and DC are tangents and $\angle ADC = 50°$.



A **[CBSE Term-II, 2015]**

Sol.



1

Given DA and DC are tangents from point D to a circle with centre O.

$\angle 1 = \angle 2 = 90°$

[radius ⊥ tangent] **1**

$\angle 1 + \angle 2 + \angle 3 + \angle 4 = 360°$ **1**

or, $90° + 90° + 50° + \angle 4 = 360°$

or, $\angle 4 = 130°$

∴ Reflex $\angle 4 = 360° - 130° = 230°$ **1**

$\angle APC = \frac{1}{2}$ reflex $\angle 4$

[angle subtended at centre]

$\angle APC = \frac{1}{2} \times 230° = 115°$ **1**

[CBSE Marking Scheme, 2015]